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Functions

1: Linear functions

DEF: Linear functions are stated as:

$$f(x) = \alpha x + q$$

The graph for the linear function is then a straight line with the slope α and it intersects the y-axis in (0,q).

Example 1:

Draw the function f(x) = ½x + 1 which is defined in D(f) =]-4;6]:

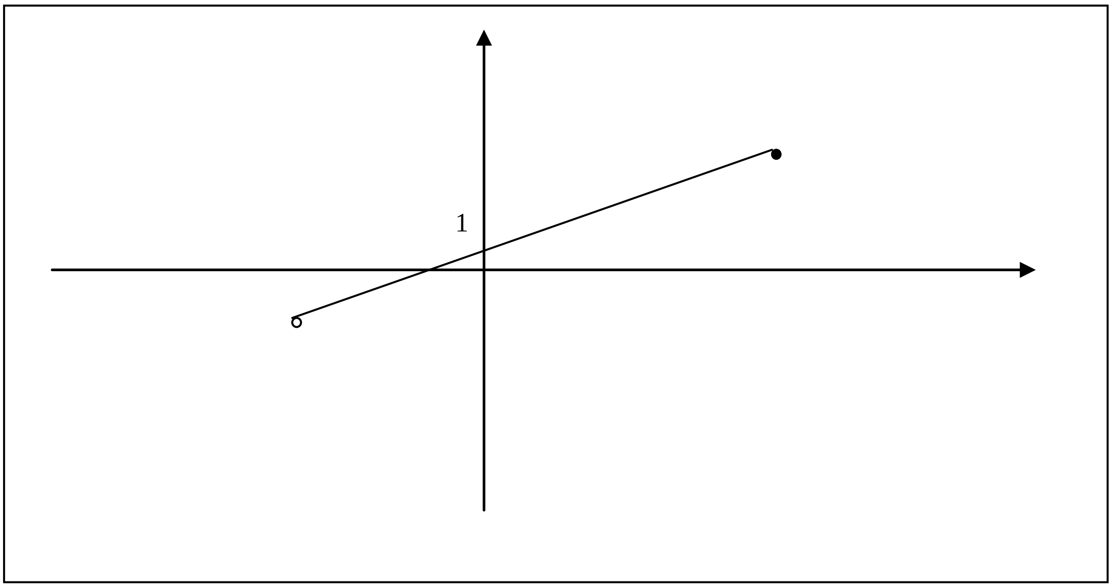

Figure 1: A linear function

Example 2:

State the linear function, where f(-2) = -12 and f(4) = 6:

Initially we will calculate the slope from the two points given of the straight line (= the linear function):

$$\alpha = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-(-12)}{4-(-2)} = 3$$

By using one of the points and the calculated slope we can now determine where the linear function intersects with the y-axis, as we know the function is stated (f(x) = αx + q):

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$$|4-2x| = x+1$$
$$\Updownarrow$$

a. $4-2x = x+1 \quad if \quad 4-2x \geq 0 \qquad \vee \qquad -(4-2x) = x+1 \quad if \quad 4-2x \langle\ 0$

$$\Updownarrow$$
$$\underline{x=1 \quad if \quad x \leq 2 \qquad \vee \qquad x=5 \quad if \quad x \rangle\ 2}$$

2. Draw both graphs to see the solution:

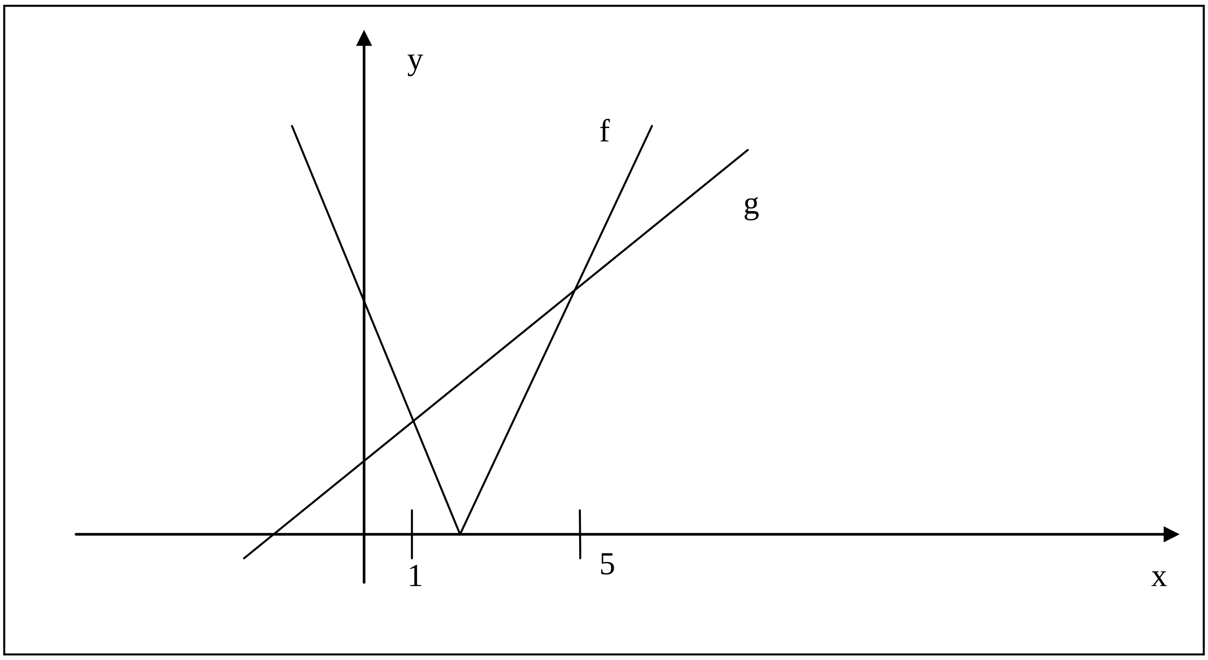

Figure 5: Solution to example 4

3. Read the solution to the inequality on the prepared figure:

   a. The solution to the inequality must then be the part of the graphs where f is located below g, therefore, the solution is:

   $\underline{\underline{Solution: x \in [1;5]}}$ (notice that both 1 and 5 is included due to the $\leq$ sign in the initial inequality

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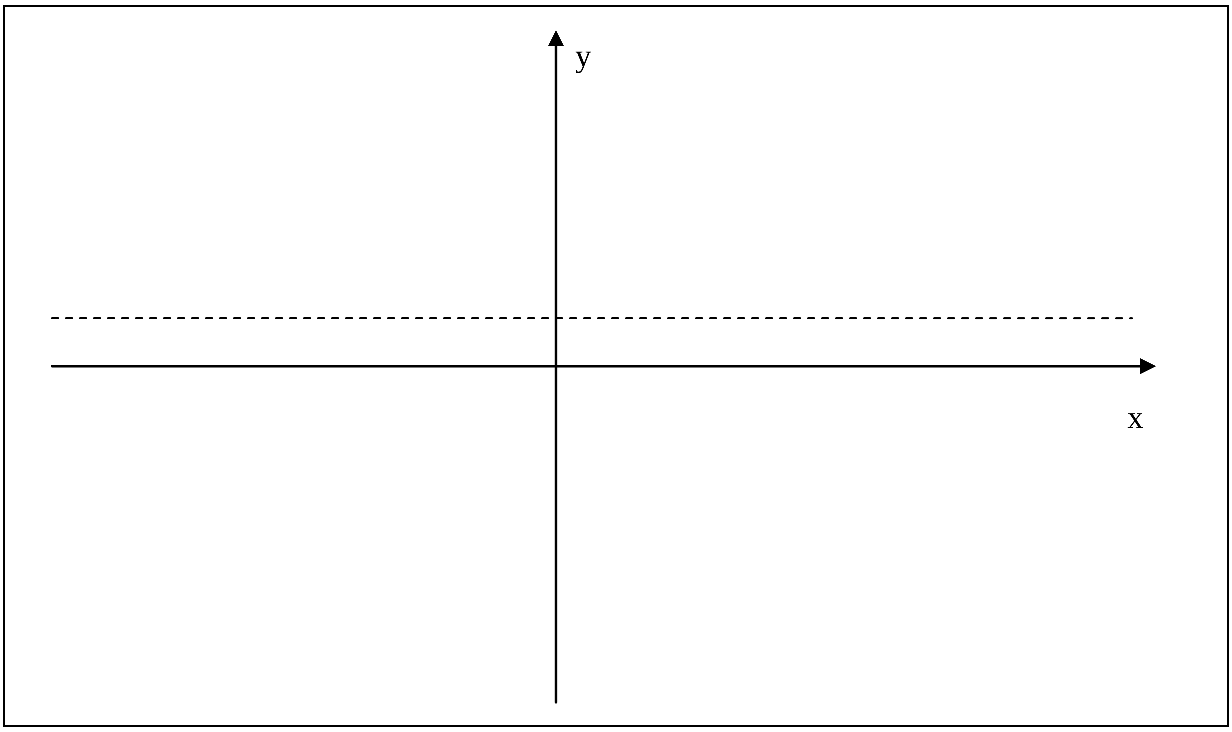

Figure 7: Vertical and horizontal asymptotes

Example 6:

Find the asymptotes for the function

$$f(x) = \frac{2x}{x-1}$$

First of all we should state the domain of the function. As the function includes a fraction one needs to make sure that the denominator never ends up as 0, therefore:

$$D(f) = R \setminus \{1\}$$

Afterwards we must investigate further what happens around x = 1 that is what happens around the number, which is *not* defined for the function. By creating a table for x and corresponding y coordinates (like above) one can conclude that:

$$f(x) \to \infty \quad as \quad x \to 1_+ \qquad or \qquad \lim_{x \to 1_+} \frac{2x}{x-1} = \infty$$

$$and$$

$$f(x) \to -\infty \quad as \quad x \to 1_- \qquad or \qquad \lim_{x \to 1_-} \frac{2x}{x-1} = -\infty$$

In other words, the line x = 1 is a vertical asymptote to the graph for the function f. Now we would like to investigate further into what happens when x tends to infinity. We do this the same way as