This is a brush up

What is a game?

A game is a formal representation of a situation in which a number of individuals interact in a setting with interdependence.

- A game can be either one of two frames:
    - Extensive form representation of a game.
    - Normal form representation of a game.

What is game theory?

Concepts for solving games.
Game theory is about fixing ideas.

Why use game theory compared to more informal theory?

Game theory allows for more in-depth analysis.

- Game theory allows us to go more in depth in the analysis of situations.

Game theory increases your understanding of strategic situations in the business sphere.

The games we study are too simplistic

- Because we're not able to work with it if it's to complex.

The rationality assumption is too demanding.

Why apply game theory to law?

1. Legal issues involve more than one person - it's not single person problems.
2. There is typically an element of conflict.
3. Game theory can help us to better predict the effects of a law or regulation
4. Game theory can help us in designing optimal contracts, laws, and regulation (changing the “rules of the game”). How can we change the rules of the game to achieve a better outcome?
5. Understanding whether social norms or institutions are better than legal rules at solving cooperation and coordination problems

3 parts:

First part: Theory and problems

- Game theory and the law, Baird et al. (1994)
- A good game theory book is Microeconomic Theory, Mas-Colell et al. (1995) chapters 7-9.
    - This book can be downloaded for free as a PDF file.

Second part: A theory of norms (Peter Høgsted)

- Order without law, Ellickson (1991)

Third part: Applications

- Papers, articles etc. Published in scientific journals. (Law and economics).

Definition

For a game with I players, the **normal form representation** Γ_N specifies for each player i a set of strategies S_i (with $s_i \in S_i$) and a payoff function $u_i(s_1, ..., s_I)$ giving the utility levels associated with the (possibly random) outcome arising from strategies $(s_1, ..., s_I)$. Formally, we write $\Gamma_N = [I, \{S_i\}, \{u_i(\cdot)\}]$

The last line:

- I = number of players.
- S_i = Sets of strategies.

Randomized choices

Randomization plays an important role in the analysis of games

- When a player randomizes over different pure strategies
- Player 1 choosing Heads with prob. 1/2 and Tails with 1/2 in Matching Pennies (Version A)

Definition

Given player i's (finite) pure strategy set S_i, a **mixed strategy** for player i, $\sigma_i : S_i \to [0, 1]$, assigns to each pure strategy $s_i \in S_i$ a probability $\sigma(s_i) \geq 0$ that it will be played, where $\sum_{s_i \in S_i} \sigma_i(s_i) = 1$

Mixed strategy Matching Pennies (Version A):

- $\sigma 1 = (\sigma 1(\text{Heads}), \sigma 1(\text{Tails})) = (1/2, 1/2)$

The sigma sign is a sign for a strategy. Instead of s1. Sigma also allows includes mixed strategies.

Simultaneous-move games

(solution concepts)

In simultaneous move games, the players chose at the same time.
The assumption of common knowledge applies here as well.
We assume that the players are rational.

- Being rational is about the expectation about the other player.

We now have a formal representation of a game

Weak Attacker	Defender: Conflict	Defender: Wait	
Conflict	−3 , −1	−1 , −2	1 − p
Wait	−2 , −1	0 , 0	

Strong Attacker	Defender: Conflict	Defender: Wait	
Conflict	0 , 0	2 , −2	p
Wait	−2 , −1	−1 , 1	

Player 1 has 1 type and player 2 has 2 types.

- Nature moves first, choosing whether we deal with the first scheme (weak attacker) or the second scheme (strong attacker).
- Of naturing has moved, the defender has the option of either conflict or wait.
- After the defender has chosen his option, the attacker has the option of either conflict or wait. What the attacker chooses decides the outcome of the game.

The defender does not know what type of attacker (player 1) is.
Nature distributes the type of attacker.

The two players (defender and the type of attacker choose at the same time and do not know what the other person chooses or will choose).

Regarding P

If p is small, we almost know for sure that it will be a weak attacker.
What if p is very close to 1?

- Then it is almost certain that it will be a strong attacker.

Weak attacker

The weak attacker's dominant strategy is to wait.
The defender wants to wait too.

Strong attacker

Common Pool Problems and Bankruptcy Law

Firms in financial distress may be subject to a “common pool problem”

The role of Bankruptcy Law and capital structure

The Creditors Game

A Debtor has borrowed $100 from two Creditors in order to finance a project

- The project has two possible outcomes: $84 (20% chance) and $122.75 (80% chance)
- The expected value of the project is $115
- The competitive rate of return for the debtors is assumed to be 0%
- The Creditors’ only decision is whether or not to monitor the Debtor
    - Monitoring costs $5 for Creditor 1 and $8 for Creditor 2
    - If only one Creditor monitors and the project fails, that Creditor can receive payment in full
    - Otherwise, the Creditors share pro rata.

In this game, monitoring brings no social benefit

The outcome which is in the Creditors’ joint interest is when neither creditor monitors

Is a combination of strategies in which neither creditor monitors a NE?

- Here we don't ask to find SPNE, because it's the same thing in a "one-shot simultaneous game"
- Yes it is.

Version A

Normal form representation of Creditors Game (low-risk debtor)

		Creditor 2	
		Don't monitor	Monitor
Creditor 1	Don't monitor	50, 50	48, 44
	Monitor	47, 48	45, 42

50% chance of failure

Suppose the project still had an expected value of $115 but only a 50% chance of success

- $146 if successful (50% chance)
- $84 if it fails (50% chance)

Is a combination of strategies in which neither creditor monitors a NE?

- No, because creditor 1 has a profitable deviation.

Version B

Normal form representation of Creditors Game (high-risk debtor)

		Creditor 2	
		Don't monitor	Monitor
Creditor 1	Don't monitor	50, 50	42, 50
	Monitor	53, 42	45, 42

At 105:

Suppose θi = 1
At 103:

At 104:

At 105:

Suppose θi = 2
At 102, 101 etc.

- The valuation won't matter because no matter what, a valuation of 2 won't meet the threshold of 104,5.

At 103:

- stating your true value (of 2) makes you strictly better off.
  - Because your value is 2, and you will only have to pay 1,5 in taxes (difference between 103 and the project value of 104,5).
- Stating 0 or 1 makes the project fail and will give you zero.
  - Because the project threshold is 104,5 and this well not be met.

At 104:

At 105:

At 103 all 2's (θi = 2) are decisive.
At 104 all 1's and 2's (θi = 1 and 2) are decisive.

Undersøg forskellen mellem weakly better off og strictly better off.

Lesson from Example E (Clark-Groves)

The government can achieve their goal if they only tax citizens who are decisive and the tax should be proportional to a citizen's stated valuation. Notice, that the costs of enforcing the law are not covered in this mechanism

A very good idea to write in an exam paper would be the possibility of the citizens meeting before stating their valuation and making a strategy - in that case, no one person would be pivotal and therefore no one person would need to pay taxes.

Mechanism Design Example (continued)

The primitives of our problem

I = 100

- They'll bargain to an efficient outcome.
- Meaning, the farmer will build a fence.
- But who will pay and how much?

Farmer's BATNA:
- The farmer's best alternative is to build a fence himself and pay the costs of $300.

Farmer's thoughts about his BATNA:
What happens to me if I don't reach a deal with the rancher?
Well, he is not liable for any crop damage...
...so I am better off building a fence at $300 than having damages worth $500.
...so if bargaining breaks, I will build a fence.
my payoff is -$300.
That's the best I can do on my own...
...so that's my BATNA.

Rancher's BATNA:
- The rancher's BATNA is to do nothing, because he will not be liable for the damages.

Rancher's thoughts about his BATNA:
What happens if I don't reach a deal with the farmer?
I am not liable for any crop damage. So I will do nothing to protect the crops.
So my payoff if we don't make a deal is $0
So that's my BATNA

The farmer's threat point is -$300, the rancher's is $0.
- Those are the payoffs that will happen if no agreement is reached.

Neither farmer nor rancher can improve their BATNA further through cooperation
There is no room for further cooperation, since total payoffs will not go up.
- So $0 gains from cooperation (or gains from trade).

End: The farmer will build the fence at a cost of $300.

Conclusion:
We have seen that:
- The initial allocation of property rights does not matter for achieving efficiency… provided there are no transaction costs.
- On the other hand, the initial allocation does affect distribution:

PAYOFF	FARMER'S RIGHT	RANCHER'S RIGHT
FARMER	$0 - $100	-$300
RANCHER	-$300 - $400	$0